

Logarithms

Question 1a.

$$3^7 = 2187$$

$$\log_3 2187 = 7$$

Question 1b.

$$8^{\frac{1}{3}} = 2$$

$$\log_8 2 = \frac{1}{3}$$

Question 2a.

$$\log_5 25 = 2$$

$$5^2 = 25$$

Question 2b.

$$\log_{216} \left(\frac{1}{6} \right) = \frac{1}{3}$$

$$216^{\frac{1}{3}} = \frac{1}{6}$$

Question 3a.

$$\log_{\sqrt{5}} 125$$

$$= \log_{5^{\frac{1}{2}}} (5)^3$$

$$\text{Using } \log_{a^y} (a^x) = \frac{x}{y}$$

$$= \frac{3}{\frac{1}{2}} = 3 \times \frac{2}{1} = \underline{\underline{6}}$$

Question 3b.

$$\log_{1/3} 81$$

$$= \log_{3^{-1}} (3)^4$$

$$\text{Using } \log_a (a^x) = \frac{x}{y}$$

$$= \frac{4}{-1}$$

$$= -4$$

Question 3c.

$$-\frac{1}{2} \log_4 (4^4)$$

$$\text{Use: } \log_a (a^x) = x$$

$$= -\frac{1}{2} \times 4$$

$$= -2$$

Question 3d

$$2 \log_5 15 - \log_5 9$$

$$= \log_5 15^2 - \log_5 9$$

$$\log_5 (15^2 / 9)$$

$$= \log_5 \left(\frac{15}{3} \right)^2$$

$$= 2 \log_5 (5)$$

$$2 \times 1$$

$$= \underline{\underline{2}}$$

Question 49

$$g(x) = \log_2 x$$

$$f(x) = 2^x$$

$$f^{-1}(x) = \frac{1}{2^x} = 2^{-x}$$

For $f(x) = 2^x$

x	-3	-2	0	1	3	2	
f(x)	$\frac{1}{8}$	$\frac{1}{4}$	1	2	8	4	

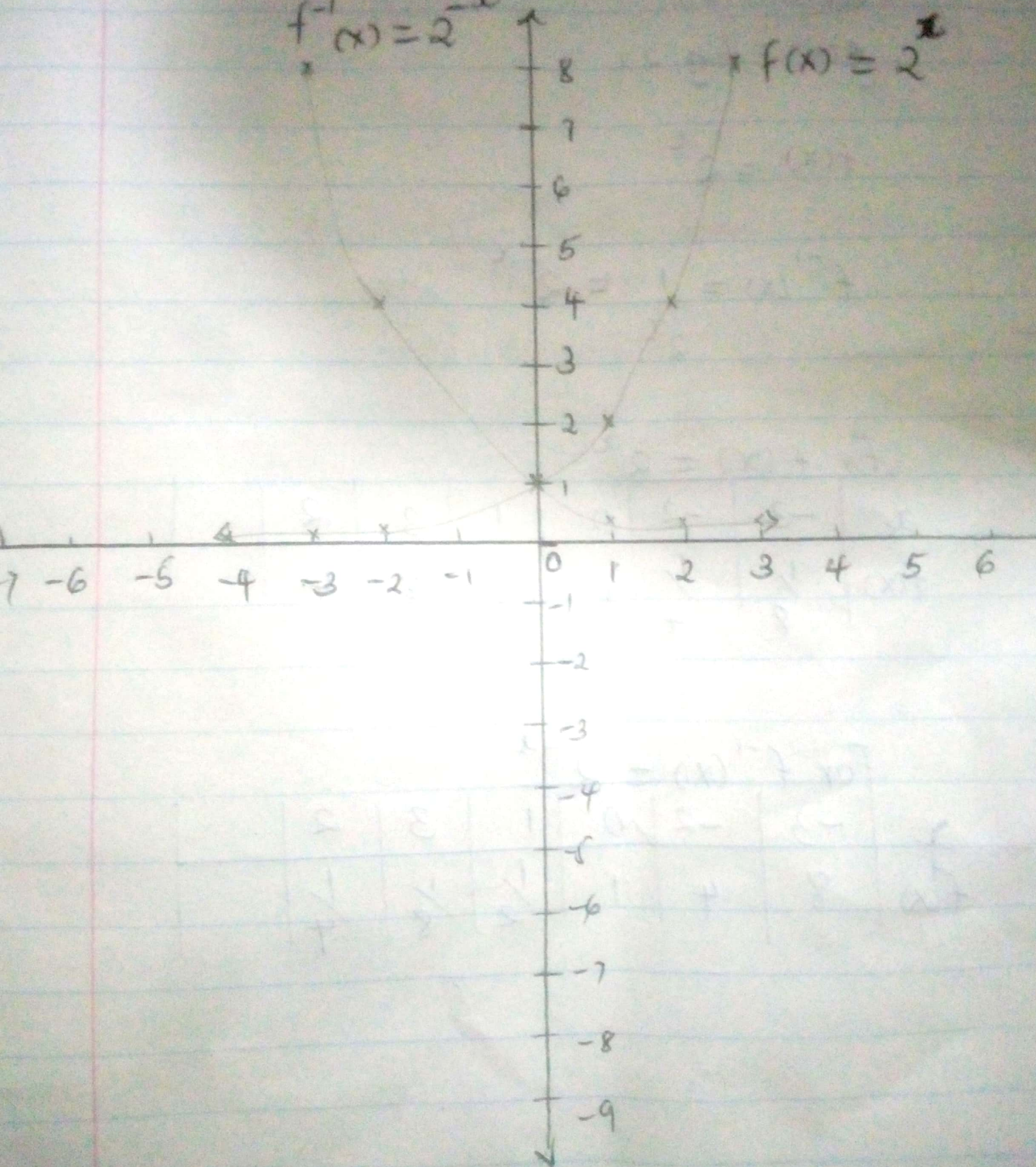
For $f^{-1}(x) = 2^{-x}$

x	-3	-2	0	1	3	2	
f ⁻¹ (x)	8	4	1	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{4}$	

Question 49 Graph.

$$f^{-1}(x) = 2^{-x}$$

$$f(x) = 2^x$$



Question 4b

$$f(x) = \log_2 x$$

$$g(x) = -\log_2 (x+2)$$

Since $g(x) = \log_2 (x+2)$, we notice $x+2=0$

This means that $x = -2$.

The vertical asymptote is $x = -2$

The Domain changes to $(-2, \infty)$ and

Range remains as $(-\infty, \infty)$

$$x+2=0, \quad c=2$$

This means that $2 > 0$, which results in an horizontal shift 2 units in the opposite direction.

Question 4c

$$g(x) = \log_2(x+2) + 2$$

$$f(x) = \log_2 x$$

for $g(x)$

$$x+2=0 \quad x=-2$$

$$\text{domain} = (-2, \infty)$$

$$\text{Vertical asymptote} = x = -2$$

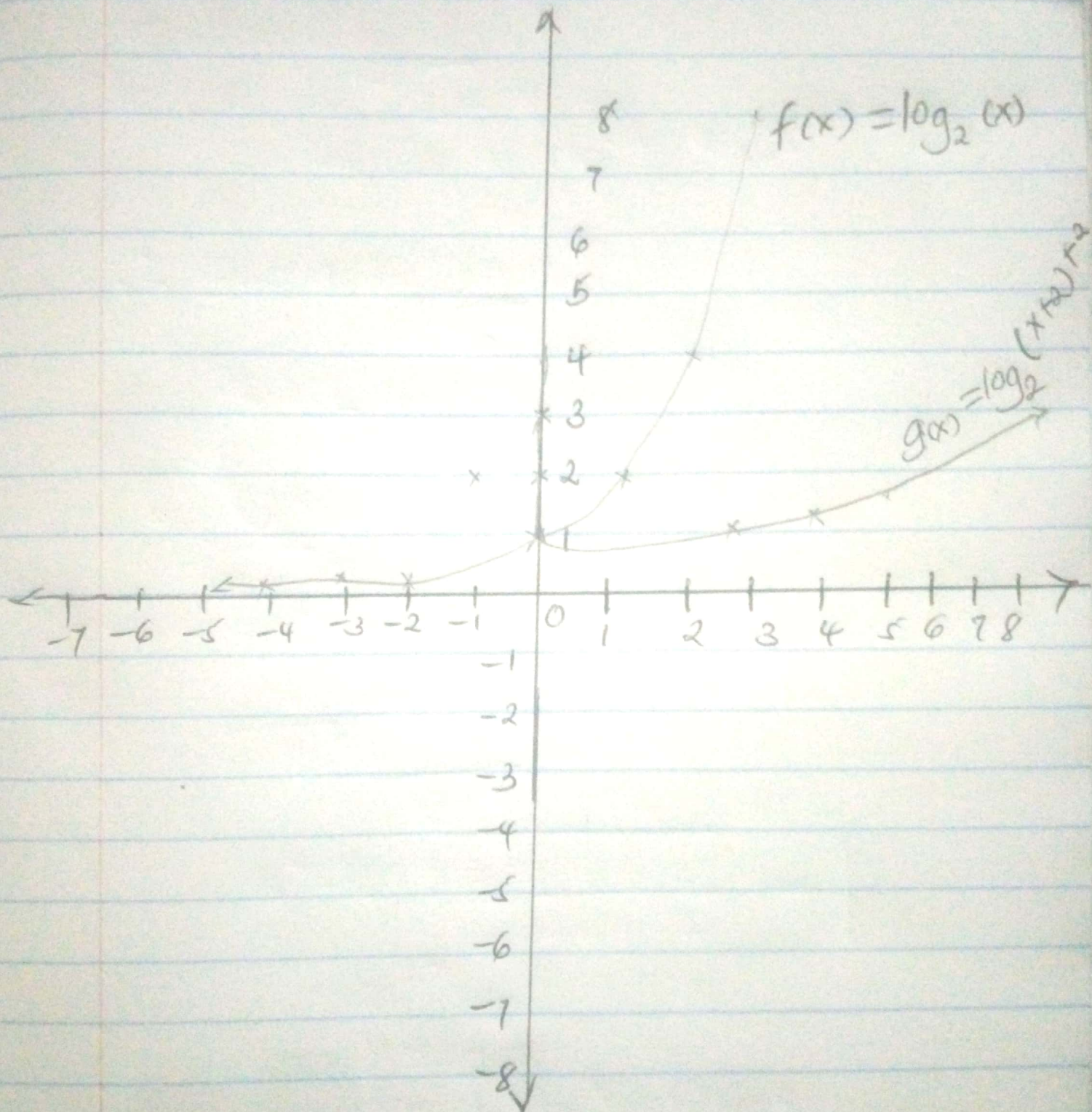
The graph passes through points
 $(-2, 2)$ $(2, -2)$

The image will shift 2 units to the left and 2 units up.

$$g(x) = \log_2(x+2) + 2$$

x	0	1	-1	3	$\frac{1}{2}$	-2	4
g(x)	3	2	2	2.7			2.8

Question 40



Question 5.

$$\|v\| = \sqrt{a^2 + b^2}$$

$$= \sqrt{4 \cdot 1^2 + 8 \cdot 2}$$

$$= \sqrt{84.05}$$

$$= 9.168$$

Hassan was right : The second earthquake was stronger than the first since it is closer to the magnitude value

Question 6a

a pH of 5.6

since they are in $\log_{10} 5.6$

$$= 0.7482$$

Question 6b:

$$\log_{10} 6.4 = \text{Audre's milk.}$$

$$= \log_{10} 3.9 - \text{Belinda's orange juice}$$

$$= \log_{10} 3.9 \div \log_{10} 6.4.$$

$$= \log_{10} (3.9/6.4)$$

$$\frac{\log_b x}{\log_b a} = \log_a x$$

$$= \log_{6.4} 3.9$$

≈ 0.7332 more acidic

Question 7.

~~4/3~~ $4^{1/3} = \text{Remaining}$

Half life carbon 14 = 5370 years.

$$A(t) = A_0 e^{kt}$$

find k

$$0.32 = 0.63 e^{k \cdot 5370}$$

$$0.51 = e^{k \cdot 5370}$$

$$\ln 0.51 = \ln e^{k \cdot 5370}$$

$$\ln 0.51 = 5370k$$

$$\frac{-0.67}{5370} = \frac{5370k}{5370}$$

$$k = -0.00013$$

$$A(t) = A_0 e^{-0.00013t}$$

$$0.63 = 1 \cdot e^{-0.00013t}$$

$$\ln 0.63 = -0.00013t$$

$$t = \frac{\ln 0.63}{-0.00013}$$

$$-0.00013$$

$$t = 3,555 \text{ years}$$

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Question 8a)

$$3^{x+4} = 243$$

$$3^{x+4} = 3^5$$

$$x+4 = 5$$

$$x = 5 - 4$$

$$x = 1$$

Question 8b

$$\frac{3}{3} (2^{x+2}) = \frac{96}{3}$$

$$2^{x+2} = 32$$

$$2^{x+2} = 2^5$$

$$x+2 = 5$$

$$x = 5 - 2$$

$$x = 3$$

Question 8c

$$5^{x+1} + 5^x = 150$$

$$(5+1) \times 5^x = 150$$

$$\frac{6}{6} \times 5^x = \frac{150}{6}$$

$$5^x = 25$$

$$5^x = 5^2$$

$$x = 2$$

Question 9a

$$x = 2 \log_2 6 - \log_2 9$$

$$= \log_2 6^2 - \log_2 9$$

$$\log_2 \left(\frac{6^2}{9} \right)$$

$$\log_2 \left(\frac{36}{9} \right)$$

$$\log_2 4$$

$$x = \log_2 2^2$$

$$x = 2$$

Question 9b

$$\log_b (x-1) + \log_b (x+2) = \log_b (8-2x)$$

$$\log_b [(x-1)(x+2)] = \log_b (8-2x)$$

$$\log_b [x^2 + 2x - x - 2] = \log_b (8-2x)$$

$$\log_b [x^2 + x - 2] = \log_b (8-2x)$$

$$x^2 + x - 2 = 8 - 2x$$

$$x^2 = 8 - 2x - x + 2$$

$$x^2 = 10 - 3x$$

$$x^2 + 3x - 10 = 0$$

$$x^2 + 5x - 2x - 10$$

$$x(x+5) - 2(x+5) = 0$$

$$(x+5)(x-2)$$

$$x+5=0 \quad x = -5$$

$$x-2=0 \quad x = 2$$

Question 4c

$$\log_7 (x+1) + \log_7 (x-5) = 1$$

$$\log_7 (x+1)(x-5) = \log_7 7$$

$$x(x-5) + 1(x-5) = 7$$

$$x^2 - 5x + x - 5 = 7$$

$$x^2 - 4x - 5 = 7$$

$$x^2 - 4x - 12 = 0$$

$$x^2 + 2x - 6x - 12 = 0$$

$$x(x+2) - 6(x+2) = 0$$

$$(x-6)(x+2) = 0$$

$$x-6=0$$

$$x=6$$

Or

$$x+2=0$$

$$x = -2$$

Question 10

10 a)

5 years ago = 4,407

10 years ago = 6,476

$$4,407 = P(1+r)^5$$

$$6,476 = P(1+r)^{10}$$

Dividing the equations

$$\approx \frac{1}{(1+r)^5} = \frac{4,407}{6,476}$$

$$\frac{1}{(1+r)^5} = 0.681$$

$$r = \sqrt[5]{\frac{6476 \times 4407^4}{4407}} - 1$$

$$r = 0.08$$

$$P(1+0.08)^{10} = 6476$$

$$P = 2,999.64$$

b)

$$A = P(1+r)^n$$

$$10,000 = 2,999.64 (1.08)^n$$

$$\frac{10,000}{2,999.64} = (1.08)^n$$

$$3.33 = (1.08)^n$$

$$\log 3.33 = n \log (1.08)$$

$$n = \frac{\log 3.33}{\log 1.08}$$

$$n = 15.63 \text{ years.}$$

Question 11

Q11

$$\beta = 10 \log \frac{I}{I_0}$$

where I_0 is 10^{-12} W/m^2

$$\therefore 50 = 10 \log \frac{I}{10^{-12}}$$

$$5 = \log \frac{I}{10^{-12}}$$

$$10^5 \neq$$

$$10^5 = 10^{\log \frac{I}{10^{-12}}}$$

$$10^5 = \frac{I}{10^{-12}}$$

$$10^5 \times 10^{-12} = I$$

$$10^{-7} = I$$

$$I = 1 \times 10^{-7} \text{ W/m}^2$$